

## Semester Courses of M.A/M.Sc. Mathematics Based on CBCS

The course of M.A/M.Sc. (Mathematics) will be spread in two years - Previous and Final. Each of which will have two semester examinations and therefore will be four semester examinations.

### Programme Specific Outcomes of M.Sc. Mathematics

1. To develop deep understanding of the fundamental axioms in mathematics and capability of developing ideas based on them.
2. To provide advanced knowledge of topics in pure mathematics particularly in Analysis and Geometry empowering the students to proceed with the area at higher level.
3. To develop understanding of applied mathematics and motivating the students to use mathematical techniques as a tool in the study of other scientific domains.
4. To encourage students for research studies in Mathematics and related fields.
5. To provide students a wide variety of employment options as they can adopt research as a career or take up teaching jobs or can get employment in banking or can go for any other profession.
6. To inculcate problem solving skills, thinking and creativity through presentations, assignments and project work.
7. To help students in their preparation (personal counselling, books) for competitive exams e.g. NET, GATE, etc.
8. To enable the students being life-long learners who are able to independently expand their mathematical expertise when needed.

### M.A./M.Sc. Previous (Mathematics) (Effective from Session 2019-2020)

The M.A./M.Sc. Previous (Mathematics) examination will consist of two semesters, called as first and second semesters. Their examinations will be held in the months of December and May respectively. In each of these semester examinations, there will be five compulsory papers. Each paper will be of three hours duration and of 5 credit (maximum marks 70), except where stated otherwise. There will be 30% internal evaluation in each paper based on:

- |                           |          |
|---------------------------|----------|
| 1. Attendance             | 10 Marks |
| 2. Class Test/ Assignment | 10 Marks |
| 3. Seminar                | 10 Marks |

### **Format of the Question Paper:-**

There will be one compulsory question consisting of 4 parts of short answer type questions based on the whole course, out of which all parts will have to be answered. Besides this, there will be 8 questions from four units (**two from each unit**), out of which 4 questions will have to be answered (**one from each unit**). Thus in all, 5 questions will have to be attempted and 9 questions will have to be set. All questions will carry equal marks, except stated otherwise.

### First Semester

S.No.	Paper	Course Code	Paper Title
1	Paper I	MAT-101	Groups and Canonical Forms
2	Paper II	MAT-102	Topology
3	Paper III	MAT-103	Differential and Integral Equations
4	Paper IV	MAT-104	Complex Analysis
5	Paper V	MAT-105	Real Analysis

### Second Semester

S.No.	Paper	Course Code	Paper Title
1	Paper I	MAT-201	Fields and Modules
2	Paper II	MAT-202	Differential Geometry of Manifolds
3	Paper III	MAT-203	Partial Differential Equations
4	Paper IV	MAT-204	Operations Research
5	Paper V	MAT-205	Fluid Dynamics

### M.A./ M.Sc. Final (Mathematics)

### (Effective from session 2020-2021)

The M.A./M.Sc. Final (Mathematics) will consist of two semesters, called as third and fourth semesters. Their examinations will be held in the months of December and May respectively. In each of these semester examinations there will be three compulsory papers and two elective papers to be selected from each group of optional papers. Each paper will be of three hours duration and of 5 credit (maximum marks 70), except where stated otherwise. There will be 30% internal evaluation in each paper based on:

1. Attendance 10 Marks
2. Class Test/ Assignment 10 Marks
3. Seminar 10 Marks

### Format of the Question Paper.

There will be one compulsory question consisting of 4 parts of short answer type questions based on the whole course, out of which all parts will have to be answered. Besides this, there will be 8 questions from four units (**two from each unit**), out of which 4 questions will have to be answered (**one from each unit**). Thus in all, 5 questions will have to be attempted and 9 questions will have to be set. All questions will carry equal marks, except stated otherwise.

### Third Semester

S. No.	Paper	Course Code	Paper Title
<b>Core Papers</b>			
1	Paper I	MAT-301	Number Theory

2	Paper II	MAT-302	Banach Spaces
3	Paper III	MAT-303	Dynamics of Rigid Bodies
	Paper IV	<b>Elective Papers</b> (Opt any one)	
4		MAT-304	Fourier Analysis and Summability Theory
5		MAT-305	General Relativity and Gravitation
6		MAT-306	Numerical Solution of Differential Equations
7		MAT-307	Advanced Topology
8		MAT-308	Hydrodynamics
	Paper V	<b>Elective Papers</b> (Opt any one)	
9		MAT-309	Discrete Mathematics
10		MAT-310	Mathematical Modelling
11		MAT-311	Complex Manifolds
12		MAT-312	Riemannian Geometry
13		MAT-313	Magneto Hydrodynamics

#### Fourth Semester

S.No.	Paper	Course Code	Paper Title
<b>Core Papers</b>			
1	Paper I	MAT-401	Measure Theory
2	Paper II	MAT-402	Hilbert Spaces
3	Paper III	MAT-403	Analytical Dynamics
	Paper IV	<b>Elective Papers</b> (Opt any one)	
4		MAT-404	Fixed Point Theory and its Application
5		MAT-405	Cosmology
6		MAT-406	Wavelet Analysis
7		MAT-407	Hydro Statics
8		MAT-408	Mathematics for Humanities (Not for Mathematics Students)
	Paper V	<b>Elective Papers</b> (Opt any one)	
9		MAT-409	Information Theory
10		MAT-410	Bio Mathematics
11		MAT-411	Contact Manifolds
12		MAT-412	Finsler Geometry
13		MAT-413	Mathematics for Life Sciences

## M.A/M.Sc. First Semester Based on CBCS (Mathematics)

### **Paper I: Groups and Canonical Form (MAT-101)**

**Total Credit-5**

**Course Objectives-**The paper of Group & Canonical form is introduced to M.Sc. classes for the study of structure of groups and properties of matrices. The main objective of group theory is that to prepare the students for further research in modern algebra

#### **Unit I**

**Groups:** Conjugacy relation. Normaliser of an element, Class equation of a finite group, Center of a group, Fundamental theorems on isomorphism of groups, Automorphisms, Inner automorphism.

#### **Unit II**

Maximal subgroups, Commutator subgroups, Composition series, Examples of Composition series and normal series. Jordan-Holder theorem, Solvable groups, Solvable subgroups, Nilpotent groups.

#### **Unit III**

External and internal direct product of groups, Cauchy's theorem for finite group, Cauchy's theorem for abelian group, Groups of order  $p^2$  and  $pq$ , Sylow's  $p$  subgroups, Sylow's first, second and third theorems. Application of Sylow's theorems to find the number of Sylow's  $p$  subgroups of a finite groups.

#### **Unit IV**

Canonical forms: Similarity of linear transformations, Invariant subspaces, Reduction to triangular forms, Nilpotent transformations, Index of nilpotency, Invariants of a nilpotent transformation, The primary decomposition theorem, Jordan blocks and Jordan forms.

#### **Books Recommended:**

1. I.N. Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul Basic Abstract Algebra (Second Edition), Cambridge University Press, Indian Edition.
3. Surjeet Singh and Qazi Zameeruddin: Modern Algebra, Vikas Publishing House. Pvt. Ltd.
4. K.B. Datta: Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi,.
5. S. Kumaresan: Linear Algebra, A Geometric Approach, Prentice Hall of India.
6. A.R. Vasishtha & A.K. Vasishtha: Modern Algebra, Krishna Prakashan Media (P) Ltd., Meerut.
7. H.K.Pathak: Abstract Algebra, Shiksha Sahitya Prakashan.

#### **Course Outcomes:**

- CO 1. Group theory covers a wide area of research in abstract algebra.
- CO 2. Sylow's theorems, group homomorphism, isomorphism etc are used to define the structure of groups as well as it is applicable in physical and chemical sciences.

CO 3. The course is very important for various competitive exams such as CSIR-NET (JRF), IAS, PCS and other teaching jobs.

**Paper II: Topology (MAT-102)**

**Total Credit-5**

**Course Objective:** A topology is a rubber sheet Geometry. Topology is concern with the properties of a geometric object that are preserved under continuous deformations such as stretching, twisting crumpling and bending but not tearing and gluing.

**Unit 1**

Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighbourhoods. Interior, exterior and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology. Neighbourhood Systems.

**Unit II**

Continuous functions and homeomorphism. First and second countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.

**Unit III**

Separation axioms  $T_0, T_1, T_2, T_3, T_4$ ; their characterizations and basic properties. Urysohn Lemma. Tietz extension theorem.

**Unit IV**

Compact sets and their properties. Finite intersection property, Bolzano Weierstrass property. Continuous functions and compactness, Sequential compactness, countable compactness and their comparison. One point compactification. Connected spaces. Connectedness on the real line. Components. Locally connected Spaces.

**Books Recommended:**

1. George F. Simmons : Introduction to Topology and Modern Analysis, Mc Graw-Hill Book Company.
2. J.L. Kelley : General Topology, Van Nostrand, Reinhold Co., New York.
3. K.D. Joshi: Introduction to General Topology, Wiley Eastern Ltd.
4. James R Munkres : Topology, Prentice Hall of India Pvt. Ltd., New Delhi.
5. Willard: General Topology Addison-Wesley, Reading.

**Course Outcome:**

- CO 1. Knot theory a branch of topology is used in biology to study the effects of certain enzymes on DNA.
- CO 2. Topology is relevant to physics in areas such as condensed matter, quantum field theory & physical cosmology.

### Paper III: Differential and Integral Equations (MAT-103)

Total Credit-5

**Course Objectives:** The objective of course is to know different method to solve ordinary differential equation and also to solve integral equations of Fredholm and Volterra type.

#### Unit I

Solution of Differential Equations in ascending and descending power series, Hypergeometric Differential Equations, Papperitz symbol, Pochhammer symbol, Hypergeometric Function, Solution of Gauss's Hypergeometric Differential Equation, Differentiation of Hypergeometric Functions.

#### Unit II

Legendre's Differential Equation, Legendre's Functions, Generating Function for  $P_n$ , Laplace Definite Integrals for  $P_n(x)$ , Orthogonal properties of Legendre's Polynomials, Recurrence Formulae, Beltrami Result, Christoffel's Expansion and Summation formulae, Rodrigue's Formula for  $P_n(x)$ .

Bessel's Differential Equation, Bessel's Functions, Generating Function for  $J_n(x)$ , Differential Equations Reducible to Bessel's Differential Equations, Orthogonality of Bessel's Functions.

#### Unit III

Integral Equations, Linear Integral Equations, Types of Linear Integral Equations, Types of Kernels, Conversion of differential equations to integral equations,  $L_2$  kernels and  $L_2$  Functions, Eigen values and eigen functions, Solution of Volterra Integral Equations by Successive Approximations and Successive Substitution Methods.

#### Unit IV

Fredholm Integral Equations of First and Second kinds, Solution of Fredholm Integral Equations by Successive Approximations and Successive Substitution Methods, Neumann Series, Volterra solution of Fredholm Integral Equation of second kind, Reduction of Volterra Integral Equation into differential equation, Reduction of Volterra Integral Equation of first kind into Volterra Integral Equation of second kind.

#### Books Recommended:

1. Differential and Integral Equations by B.P. Parashar
2. Series Solution and Special Functions by V. S. Verma
3. Fundamentals of Integral Equations by V. S. Verma
4. Integral Equations and Boundary Value Problems by M D Raisinghania
5. Integral Equations by Shanti Swarup and Shiv Raj Singh
6. Linear Integral Equations by R. P. Kanwal

**Course Outcomes:** On completion of this course students will also able to use Fourier transform for solving a wide range of differential and Integral equations:

- CO 1. Formulate and solve initial and boundary value problems for the heat and wave equations in spherical and cylindrical coordinates.
- CO 2. Solve linear Volterra and Fredholm integral equations using appropriate methods.

CO 3. Understand the relationship between integral and differential equations and transform one type into another.

**Paper IV: Complex Analysis (MAT-104)**

**Total Credit-5**

**Course Objective:** The main objectives are

1. To understand power series and its region of convergence
2. To study analytic continuation
3. To understand conformal mapping and related results
4. To understand Schwarz's lemma and related results for further study

**Unit I**

Conformal Mapping, Mobius (Bilinear) transformations: involving circles and half-planes, fixed point, cross ratio, Transformations  $w=z^2$ ,  $w = \tan^2(z/2)$ , Univalent function and its properties. Many valued functions and its properties.

**Unit II**

Power series and its convergence. Analyticity of power series, singularity of power series, Gamma function. Zeta Function

**Unit III**

Maximum-modulus theorem. Schwarz's lemma. Hadamard's three-circles theorem. Borel-Cartheodory theorem. Phragmen- Lindelof theorem.

**Unit IV**

Analytic continuation. Uniqueness of analytic continuation. Power series method of analytic continuation. Natural boundary.

**Books Recommended:**

1. E.C. Titchmarsh: Theory of Functions, Oxford University Press, London.
2. Mark J. Ablowitz and A.S. Fokas: Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
3. R.V. Churchill & J.W. Brown. Complex Variables and Applications, 5<sup>th</sup> Edition McGraw-Hill, New York, 1990.
4. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand & Co., New Delhi.
5. S. Ponnusamy, Foundation of Complex Analysis, Narosa Publication

**Course Outcomes:** After the course the students will be able to

- CO 1. understand the basics of this course
- CO 2. understand the use of this course in different field of mathematical Analysis
- CO 3. think and develop new ideas in this course
- CO 4. get benefit of this course in various national and international competitive examinations

**Course Objectives:** The paper of Real Analysis is introduced to M.Sc. classes for the study of functions of bounded variation, Riemann-Stieltjes integrals, pointwise convergence, uniform convergence and power series. The main objectives of real analysis is that to prepare the students for further research in analysis and deferential geometry.

**Unit I**

Functions of Bounded Variation and some properties of function of bounded variation, Lipschitz condition and function. Variation function and The Jordan Decomposition theorem.

**Unit II**

Definition and Existence of Riemann- Stieltjes integrals. Properties of the integral, integration and differentiation, the first and second mean value theorem, the fundamental theorem of integral calculus, change of variable and Integration by parts for Riemann- Stieltjes. Relation between Riemann and Riemann- Stieltjes integral. Riemann- Stieltjes integrals and bounded variation.

**Unit III**

Sequences of functions of real numbers. Pointwise convergence and uniform convergence. Cauchy Criterion of uniform convergence,  $M_n$  test, Weierstrass M- test, everywhere continuous but nowhere differentiable functions. Dini's criterion of uniform convergence. Uniform convergence and continuity. Continuity of limit function. Uniform convergence and Riemann stieltjes integration, Uniform convergence and differentiation.

**Unit IV**

Abel's and Dirichlet's tests for uniform convergence. Connections between Riemann- Stieltjes integrals, uniform convergence and bounded variation. Curves, Rectifiable curves, Additive and Continuity properties of arc length. Rearrangements of terms of a series, Dirichlet's and Riemann's theorem. Power series, uniqueness theorem for power series, Abel's and Tauber's theorem for power series.

**Books Recommended:**

1. Walter Rudin: Principles of Mathematical Analysis (3rd edition), McGraw-Hill, Kogakusha, 1976 International Student Edition.
2. H. L. Royden: Real Analysis, Macmillan Pub. Co. Inc. New York, 4th Edition, 1993.
3. Richard Johnson Baugh: Foundation of Mathematical Analysis.
4. H.K.Pathak: Real Analysis, Shiksha Sahitya Prakashan.
5. Apostol: Mathematical Analysis, Narosa Publishing House.

**Course Outcomes:**

- CO 1. In mathematics real analysis is the branch of mathematical analysis.
- CO 2. Students will be able to effectively write mathematical solutions in a clear and concise manner.
- CO 3. Also demonstrate an intuitive and computational understanding of functions of bounded variation, Riemann-Stieltjes integrals, pointwise convergence, uniform convergence and power series.

CO 4. Real analysis covers a wide area of research in analysis and differential geometry. This course is useful in various competitive exams like CSIR-NET, IAS, PCS.

**M.A/M.Sc. Second Semester Based on CBCS (Mathematics)**

**Paper I: Fields and Modules (MAT-201)**

**Total Credit-5**

**Course Objective:** The main objectives are

1. To understand extension field and related results, Algebraic and transcendental extension, splitting fields
2. To understand normal extensions, perfect field, finite fields
3. To understand Galois group
4. To understand Modules, cyclic modules and related results

**Unit I**

Field theory: Extension Fields. Algebraic and transcendental extensions. Splitting Field. separable and inseparable extensions.

**Unit II**

Normal extension. Perfect Fields. Finite Fields. Automorphisms of extensions.

**Unit III**

Galois group. Fundamental theorem of Galois theory. Construction with ruler and compass. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

**Unit IV**

Modules, Cyclic modules. Simple modules . Semi-simple modules. Schuler's lemma. Free modules. Noetherian and artinian modules. Hilbert basis theorem.

**Recommended Books:**

1. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul : Basic Abstract Algebra (Second Edition), Cambridge University Press, Indian Edition.
3. Surjeet Singh and Qazi Zameeruddin: Modern Algebra, Vikas Publishing House. Pvt. Ltd.
4. K.B. Datta : Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi.
5. S. Kumaresan : Linear Algebra, A Geometric Approach, Prentice Hall of India.
6. A.R. Vasishtha & A.K. Vasishtha : Modern Algebra, Krishna Prakashan Media (P) Ltd., Meerut .
7. H.K.Pathak: Abstract Algebra, Shiksha Sahitya Prakashan.

**Course Outcomes:** After the course the students will be able to

- CO 1. understand the basics of this course
- CO 2. understand the applications of this course in different field of Science and Technology

CO 3. think and develop new ideas in this subject

CO 4. get benefit of this course in various national and international competitive examinations

## **Paper II: Differential Geometry of Manifolds (MAT-202)**

**Total Credit- 5**

**Course Objective:** The paper of Differential Geometry of Manifolds is introduced to M.Sc. classes for the study of Tensor Algebra, Differentiable manifold, Differentiable functions, Differentiable curves, Tangent space, Vector fields, Lie bracket, Covariant differentiation, Torsion, Curvature, Lie derivative, Riemannian Manifold, Exterior algebra and Submanifolds & Hypersurfaces. The main objective of Differential Geometry of Manifolds is that to prepare the students for further research in analysis of differential geometry.

### **Unit I**

Tensor Algebra: Contravariant and covariant vector. Tensor product of vector spaces, tensor, contravariant, covariant and mixed tensor of second order. Tensor of type  $(r, s)$ , tensor product of tensors. Algebraic operations, symmetric and skew symmetric tensors, contraction.

### **Unit II**

Definition and examples of differentiable manifold. Differentiable functions. Differentiable curves. Tangent space. Vector fields. Lie bracket. Invariant view point of connections. Covariant differentiation.

### **Unit III**

Torsion. Curvature. Parallelism. Difference tensor of two connections. Lie derivative. Riemannian Manifold. Riemannian connection. Riemannian curvature tensor and Ricci tensor. Identities of Bianchi. Sectional curvature and Schur's theorem.

### **Unit IV**

Exterior product of two vectors. Exterior algebra of order  $r$ . Exterior derivative. Cartan's structural equations. Submanifolds and Hypersurfaces. Normals. Gauss's formula. Weingarten equations.

### **Books Recommended:**

1. R. S. Mishra, A Course in Tensors with Applications to Riemannian Geometry, Pothishala, Allahabad, 1965.
2. Y. Matsushima, Differentiable Manifolds, Marcel Dekker, 1972.
3. B. B. Sinha, An Introduction to Modern Differential Geometry, KalyaniPrakashan, New Delhi, 1982.
4. Y. Talpiert, Differential Geometry with applications to Mechanics and Physics, Marcel Dekkar Inc. 2001.
5. N.J. Hicks, Notes on Differential Geometry, D. Van Nostrand Inc., 1965.
6. U.C.De and A.A.Shaikh, Differential Geometry of Manifolds, Narosa Publishing House, New Delhi 2007.
7. K.S.Amur, D.J.Shetty and C.S.Bagewadi, An Introduction to Differential Geometry, Narosa Publishing House, New Delhi 2010.

8. S. Shahshahani, An Introductory Course on Differentiable Manifolds, Dover Publication Inc. New York, 2016.

**Course Outcome:**

- CO 1. In mathematics Differential Geometry of Manifolds is the branch of differential geometry.  
CO 2. Students will be able to demonstrate an intuitive and computational understanding of Tensor Algebra, Differentiable manifold, Riemannian Manifold, Exterior algebra and Submanifolds & Hypersurfaces.  
CO 3. Differential Geometry of Manifolds covers a wide area of research in differential geometry. It is also used in physical sciences and Cosmology.

**Paper III: Partial Differential Equations (MAT-203)**

**Total Credit-5**

**Course Objective:** The objective of this paper is:

- (1) To introduce students to partial differential equation
- (2) To introduce students to how to solve linear partial differential equation with different methods
- (3) To study about basic definition of Partial differential equation, classification and geometrical interpretation

**Unit I**

Origin of first order partial differential equations, Lagrange's solution of first order linear partial differential equation.

**Unit II**

Non-linear partial differential equations of the first order, Cauchy's method of characteristics, Charpit's method and Jacobi's method.

**Unit III**

Origin of second order partial differential equations, Higher order partial differential equations with constant coefficients, Equations with variable coefficients, Classification of second order partial differential equations, Canonical forms.

**Unit IV**

Solution of non-linear second order partial differential equations by Monge's method, Method of separation of variables for solving Laplace, wave and diffusion equations.

**Books Recommended:**

1. V.S.Verma : A Text Book of Partial Differential Equations
2. A.R. Forsyth : A Treatise on Differential Equations
3. I.N. Sneddon : Elements of Partial Differential Equations.
4. LC Evans, Partial Differential Equations, AMS, 1998
5. M.D. Rai Singhania: Advanced Differential Equations, S. Chand.
6. Donald Greenspan: Introduction to Partial Differential Equations, Dover publications, New York.

7. Peter V. O'Neil: Beginning Partial Differential Equations, Wiley.

**Course Outcome:**

- CO 1. The students would be able to describe real- world system using partial differential equations
- CO 2. Students can solve first order partial differential equations and second order partial differential equations using different methods
- CO 3. They can determine the existence and uniqueness of solutions of Partial differential equations

**Paper IV: Operations Research (MAT-204)**

**Total Credit-5**

**Course Objectives:** Operations research is included in M.Sc. classes due to its wide application in our daily life. Operations research is an important course in applied mathematics because it is very useful in Industry, banking, Defense sector, Multinational companies etc to optimize their performance.

**Unit I**

Inventory Control: Introduction, Classification of Inventory, Economic parameter associated with inventory problems, Deterministic and Probabilistic models with without lead time.

**Unit II**

Sequencing Problems :Assumptions for seqncing problem. Processing n jobs on two machines, n jobs on three machines, 2 jobs on m machines, Problem of Replacement, Individuals and Group replacement policies.

**Unit III**

Network analysis: Basic concepts and definition. Network drawing and analysis Critical path method. Labelling method. Methods based on time estimates to find critical path. Concept of slack and float. Resource levelling and time-cost trade-off analysis. Time-cost optimization procedure. Project crashing. PERT. Requirements for application of PERT technique. Practical limitations in using PERT. Differences in PERT and CPM.

**Unit IV**

Non-Linear Programming:Introduction and defintions. Formulation of non-Linear programming problems, General non-linear programming problems. Kuhn-Tucker conditions, Lagrangian Method, Constrained optimization with equality constraints. Constrained optimization with inequality constaints. Saddle point problems Saddle points and NLPP. Wolfe's and Beale's method to solve Quadratic Programming problem.

**Books Recommended:**

1. S.D. Sharma: Operations Research, Kedar Nath Ram Nath & Company.

2. S.S. Rao: Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.
3. J.K. Sharma: Operations Research – Theory and Applications, Macmillan India Ltd.
4. H.A. Taha: Operations Research – An Introduction, Macmillan Publishing Co., Inc., New York.
5. Kanti Swarup, P.K. Gupta, Man Mohan: Operations Research, Sultan Chand and sons, New Delhi.
6. B.S. Goel, S.K. Mittal: Operations Research, Pragati Prakashan, Meerut.
7. P.K. Gupta, D.S. Hira: Operations Research – An Introduction, S. Chand & Company Ltd., New Delhi.

### **Course Outcomes:**

- CO 1. Due to its applicability in different sectors Operations research becomes very useful course in research field.
- CO 2. After studying this course students may do their research work in different topics like Game theory, Job sequencing, Network analysis, dynamical programming etc.
- CO 3. Most of the companies hire OR technician to get maximum output of company.

### **Paper V: Fluid Dynamics (MAT-205)**

**Total credit-5**

**Course Objective:** The paper of Fluid Dynamics is introduced to M.Sc. classes for the study of Fluid motion, Lagrangian and Eulerian methods, Euler's and Lagrange's Equation of continuity and equation of motion, Newton's law of viscosity Navier-Stokes equations of motion, Steady viscous flow between parallel planes. The main objective of Fluid Dynamics is that to prepare the students for further research in applied mathematics.

#### **Unit I**

Fluid motion, Lagrangian and Eulerian methods, Stream Lines. Path lines and streak lines. Velocity potential. Vorticity Vector. Irrotational and rotational motions. Boundary surfaces. Euler's and Lagrange's Equation of continuity. Forms of equation of continuity in different coordinate system. Symmetrical forms of equation of continuity. Lagrange's and Euler's equations of motion. Pressure equation. Bernoulli's theorem.

#### **Unit II**

Newton's law of viscosity. Kinds of fluids. Nature of stress. Stress components in a real fluid. Symmetry of stress tensor. Transformation of stress components. Stress invariants. Relations between Cartesian components of stress. Rate of strain quadric. Principal stresses. Stoke's law of viscosity. Relations between stress and rate of strain.

#### **Unit III**

Navier-Stokes equations of motion, Steady viscous flow between parallel planes. Laminar flow between parallel plates(walls). Plane Couette flow. Plane Poiseuille flow. Hagen-Poiseuille flow. Steady flow through a tubes of uniform circular cross-sections.

## Unit IV

Steady flow between concentric rotating cylinders. Laminar steady flow between two coaxial circular cylinders. Diffusion of vorticity. Energy dissipation due to viscosity. Reynolds number. Significance of Reynolds number.

### Books Recommended:

1. J.K. Goyal and K.P. Gupta: Fluid Dynamics, Pragati Prakashan, Meerut, 2017
2. N. Curle and H. J. Davis: Modern Fluid Dynamics, D. Van Nostrand Company Ltd. London, 1968.
3. G.K. Batchelor: An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, 2000.

### Course Outcome:

- CO 1. In mathematics Fluid Dynamics is the branch of applied mathematics.
- CO 2. Students will be able to effectively write mathematical solutions in a clear and concise manner.
- CO 3. Also demonstrate an intuitive and computational understanding of Fluid motion, Lagrangian and Eulerian methods, Euler's and Lagrange's Equation of continuity , Newton's law of viscosity Navier-Stokes equations of motion, Steady viscous flow between parallel planes.
- CO 4. Fluid Dynamics covers a wide area of research in applied mathematics. This course is useful in various competitive exams like CSIR-NET, IAS, and PCS.

## M.A/M.Sc. Third Semester Based on CBCS (Mathematics)

**Paper I: Number Theory (MAT-301)**

**Total Credit-5**

**Course Objective:** The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand Division algorithm, Common divisor, Greatest common divisor, Euclid's lemma, Relatively prime, Euclidian Algorithm, Fundamental theorem of arithmetic, Euclid's theorem, Congruence's, function  $\phi$  Euler's, Prime modules and Cryptography, Quadratic reciprocity, Fibonacci numbers.
3. To prepare the students for further research in Number Theory and coding theory.

### **Unit I**

Divisibility: Some basic terms and properties, Division algorithm, Common divisor, Greatest common divisor (gcd), Theorems on gcd, Euclid's lemma, Relatively prime, Euclidian Algorithm, least common multiple (lcm), Theorems on lcm, Fundamental theorem of arithmetic, Euclid's theorem.

### **Unit II**

Congruences: Theorems of congruences, Residue and complete residue system, Reduced residue system, Euler's  $\phi$  function, Euler's theorem, Fermat's theorem, Wilson theorem, Converse of Wilson theorem, Solutions of congruences, Degree of congruence, Chinese remainder theorem, Method of solution of congruences.

### **Unit III**

Prime modules and Cryptography: Prime modules, Power residues, Number theory from algebraic point of view, Introduction of cryptography, Some simple cryptosystems, Enciphering matrices.

### **Unit IV**

Quadratic reciprocity: Quadratic residues, Gauss lemma, Gaussian reciprocity law, Jacobi symbol, Greatest integer function, Arithmetic function, Multiplication of arithmetic functions, Moebius function, Moebius inversion formula, Converse of Moebius inversion formula, Recurrence functions, Fibonacci numbers

### **Books recommended:**

1. Niven and Zuckermann: An Introduction to the theory of numbers, Wiley Eastern Ltd.
2. Ireland & Rosen, A Classical Introduction to Modern Number Theory, Springer
3. Tom Apostol, Introduction to Analytic Number theory, Narosa Publications, New Delhi
4. Delfs, H., Knebl, H., Introduction to Cryptography, Springer.
5. Koblitz, N., Algebraic Aspects of Cryptography, Springer.
6. Serre, J.P., A Course in Arithmetic, Springer.
7. Cassels, J.W.S., Frolich, A., Algebraic Number Theory, Cambridge

**Course Outcome:** After studying this course the student will be able to

CO 1. understand the basic of this course and think & develop new ideas in this course.

- CO 2. know Division algorithm, Common divisor, Greatest common divisor, Euclid's lemma, Relatively prime, Euclidian Algorithm, Fundamental theorem of arithmetic, Euclid's theorem, Congruences, function  $\phi$  Euler's, Prime modules and Cryptography, Quadratic reciprocity, Fibonacci numbers.
- CO 3. cover a wide area of research in in Number Theory and coding theory.

**Paper II: Banach Spaces (MAT-302)**

**Total Credit-5**

**Course Objective:** Banach spaces originally grew out of the study of function spaces. The theory of Banach spaces developed in parallel with the general theory of linear topological spaces. These theories mutually enriched one another with new ideas and facts. A Banach space is a vector space with a metric that allows the computation of a vector length and distance between vectors and is complete in the sense that Cauchy sequence of a vectors always converges to a well-defined limit that is within the space.

**Unit I**

Normed linear spaces, Banach spaces, their examples including  $\mathbb{R}^n, \mathbb{C}^n, l_p(n), 1 \leq p < \infty, c_0, c, l_p, 1 \leq p < \infty, P[a,b], C[a,b]$ . Joint continuity of addition and scalar multiplication. Summable sequences and completeness. Subspaces, Quotient spaces of normed linear space and its completeness.

**Unit II**

Continuous and bounded linear operators and their basic properties. Normed linear space of bounded linear operators and its completeness. Equivalent norms. Finite dimensional normed spaces and compactness.

**Unit III**

Isometric isomorphism, Topological isomorphism. Riesz Theorem, Open mapping theorem and its simple consequences. Product normed space. Closed graph theorem. Uniform boundedness. Banach-Steinhaus theorem. Adjoint of bounded Linear operators.

**Unit IV**

Bounded linear functionals Dual spaces. Form of dual spaces  $(\mathbb{R}^n)^*, (\mathbb{C}^n)^*, c_0^*, l_1^*, l_p^*, 1 < p < \infty$ . Hahn- Banach theorem for real and complex normed linear spaces and its simple consequences. Embedding and Reflexivity. (1credit)

**Books Recommended:**

1. P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.
2. B. Choudhary and S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
3. I.J Maddox: Functional Analysis, Cambridge University Press (1970).
4. B.V.Limaye: Functional Analysis, New Age International Publications, New Delhi.

5. K. Chandrashekhara Rao. Functional Analysis, Narosa Publishing House, New Delhi
6. W.Rudin: Functional Analysis, TMH, New Delhi H.K.Pathak: Functional Analysis with Applications, Siksha Sahitya Prakashan, Merrut

**Course Outcomes:**

CO 1. Banach space is a part of functional analysis. Functional analysis is the backbone of modern applied and computational mathematics.

**Paper III: Dynamics of Rigid Bodies (MAT-303)**

**Total Credit-5**

**Course Objectives:**

1. To teach students the basic principles underlying the dynamics of rigid bodies in planar and 3D motion.
2. To educate students to identify, formulate and solve problems in rigid body dynamics.
3. To introduce students to the concepts of work-energy and impulse-momentum for rigid body systems.

**Unit I**

Rigid Body, properties of rigid body and its motion, impressed and effective forces, finite and impulsive forces, D' Alembert's principle, general equation of motion of a rigid body from D'Alembert's principle, motion of the centre of inertia, motion about centre of inertia, Application of D'Alembert's principle to impulsive forces,

**Unit II**

Motion about a fixed axis: moment of effective forces about the axis of rotation, kinetic energy, equation of motion, compound pendulum, centre of suspension, minimum time of oscillation of compound pendulum, reaction of the axis of rotation, motion about a fixed axis (impulsive forces).

**Unit III**

Motion of a rigid body in two dimensions under finite forces: Equation of motion, Kinetic energy, moment of momentum in two dimensions, motion of solid sphere down an inclined plane, slipping of rods, motion of solid sphere down an inclined plane when rolling and sliding are combined, Motion of circular disc, determination of sliding at the point of contact during relative motion of two bodies in contact, motion of one sphere on the other which is fixed, motion of solid cylinder inside a hollow cylinder, motion of one body on the other when the lower body is free to turn about its axis, motion of one body on another when both bodies are free to move.

**Unit IV**

Motion in two dimensions under impulsive forces, Moving axes and the fixed axes, Rotation of a vector in 2D and 3D space, motion of a particle in rotating space, motion of a rigid body in rotating frame, effects of earth rotation, free particle motion relative to earth, effects of coriolis force on some natural events, Eulerian approach to rigid body motion, Euler dynamical equations of motion for finite and impulsive forces, Kinetic energy of a rigid body about a fixed point, Eulerian angles and geometrical relations, instantaneous axis of rotation, invariable line, locus of invariable line.

### **Books Recommended:**

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies, Macmillan India Ltd., 1982.
2. A.S. Ramsey: Dynamics Part-II, The English Language Book Society and Cambridge University Press, 1972.
3. J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book Company, 1982.
4. P.P. Gupta, G.S. Malik: Rigid Dynamics I, Krishna Prakashan Media (P) Ltd.2008.
5. Naveen Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi.

### **Course outcomes:**

- CO 1. Students will have an understanding of Newtonian-Eulerian physics and basic equations underlying kinematics and kinetics of rigid bodies in 2D and 3D motion.
- CO 2. Students will acquire an ability to identify and effectively account for kinematic constraints such as rolling and/or sliding, and their kinetic consequences.
- CO 3. Students will be able to apply and combine the appropriate principles of Eulerian Physics to the solution of problems.
- CO 4. Students will be able to calculate the principal moments of inertia for arbitrary rigid bodies.
- CO 5. Students will acquire an understanding of work-energy principles as applied to rigid bodies in 2D and 3D motion.
- CO 6. Students will be able to evaluate the kinetic energy of rigid bodies as well as the potential energy associated with gravity.
- CO 7. Students will demonstrate an understanding of conservation laws for momentum and energy.

### **Paper IV: Fourier Analysis and summability Theory (MAT-304)**

**Total Credit-5**

**Course Objectives:** The main objectives are:

1. To study Riemann-Lebesgue theorem, Dini's test, Dela vallee-Pouesin's test, summation of series by arithmetic means.
2. To understand summability of Fourier series, Weierstrass approximation theorem and related results.
3. To study Bessel's inequality, Riesz-Fischer theorem, properties of Fourier coefficients, Conto's lemma, Riemann's First and second theorem.
4. To study special methods of summation.

### **Unit I**

Convergence problem, Dirichlet's integral, Riemann-Lebesgue Theorem, Convergence tests, Dini's, Jordan's, de la Vallee-Poussin's tests and their inter-relations. Summation of series by arithmetic means

## Unit-II

summability of Fourier series, Fejer's theorem, Weierstrass's approximation theorem, Almost everywhere summability, The Fej\_er-Lebesgue theorem, A continuous function with a divergent Fourier series, Order of partial sums, Integration of Fourier series, Convergent trigonometric series need not be a Fourier series, Parseval's theorem.

## Unit—III

Functions of the class  $L^2$ : Bessel's inequality, Parseval's theorem for continuous functions, The Riesz- Fischer theorem, Properties of Fourier coefficients, Uniqueness of trigonometric series, Cantor's lemma, Riemann's First and second theorems.

## Unit-IV

Special methods of summation: Norlund means, Regularity and Consistency of Norlund means, Inclusion, Equivalence, Euler's means, Abelian means, Riesz's typical means. Arithmetic means: Holder's means, simple theorems concerning Holder summability, Cesaro means, means of non-integral orders, simple theorems concerning Cesaro summability, Cesaro and Abel summability, Cesaro means as Norlund means, Tauberian theorems for Cesaro summability.

### Recommended Books:

1. E.C. Titchmarsh: A Theory of Functions, Oxford University Press, 1939.
2. A Zygmund: Trigonometric series Vol. I, The University Press, Cambridge 1959
3. G. H. Hardy: Divergent series, The Clarendon Press, Oxford, 1949.

**Course Outcomes:** After the course the students will be able to

- CO 1. understand the basics of Fourier series and summability theory.
- CO 2. to understand the use of this course in different field of mathematical Analysis.
- CO 3. to think and develop new ideas in this field.

## Paper IV: General Relativity and Gravitation (MAT-305)

**Total Credit-5**

**Course Objective:** The objective of this paper is to study new theory of gravitation introduced by Albert Einstein in 1915. This theory can be used to solve many rigorous problems of universe.

### Unit I

Space time, Curved Space time, Riemannian metric, Riemannian curvature tensor, Conformal curvature tensor, Parallel transport and Geodesic, Geodesic Deviation., Lie derivative.

### Unit II

Introduction to General Relativity, Principle of Equivalence, Principle of General covariance, Mach's Principle, Newtonian approximation of equation of motion. Einstein's field equation, Gravitational field in empty space

### Unit III

Schwarzschild exterior solution, Singularities in Schwarzschild line element, Kruskal-Szekers coordinate, Isotropic form of Schwarzschild exterior line element, Birkhoff's theorem

### Unit IV

Planetary orbits. The advanced of perihelion, Bending of light rays in gravitational field, The gravitational red-shift of spectral lines, Kepler's Law in General Relativity, Energy momentum tensor, Formula for energy momentum tensor for perfect fluid, Vaidya metric.

### **Books Recommended:**

1. K. D. Krori : Fundamentals of Special and General Relativity; PHI Publication, 2010.
2. S. R. Roy and Raj Bali : Theory of Relativity; Jaipur Publishing House, 2008.
3. Steven Weinberg : Gravitation and Cosmology : Principles and applications of General Relativity; Wiley Publ.,2005.
4. J. V. Narlikar : An Introduction to Relativity; Cambridge University Press, 2010.
5. I.B. Khriplovich : General Relativity; Springer Science + business media, 2005.

### **Course Outcomes:**

- CO 1. After the completion of this course, the students will be able to understand the beauty of general Relativity theory as a bridge between physics of Universe with their geometry.
- CO 1. Application of general relativity to describe the evolution of universe and most of the cosmic problems.
- CO 2. Students will be able to know the importance of this theory in solving the problem of Universe with the geometry and geometric structures.

### **Paper IV: Numerical Solution of Differential Equations (MAT-306)**

**Total Credit-5**

**Course Objectives:** The objective of this paper is

1. To study numerical solution of differential equations and system of non linear equations
2. To introduce students to Runge- Kutta method for time dependent problem
3. To introduce students to boundary value problems using finite difference, finite element and spectral methods
4. To study fix point iteration and Newton's method for solving systems of non linear equations

### **Unit I**

Numerical Solution of parabolic partial differential equations (PDE) in one space: two and three levels explicit and implicit differences schemes. Convergence and stability analysis.

### **Unit II**

Numerical solution parabolic PDE of second order on two spaces dimension: Implicit methods, alternating direction implicit (ADI) methods. Non-linear initial BVP (boundary valued problems). Differences schemes for parabolic PDE in spherical and cylindrical coordinate systems in one dimension.

### **Unit III**

Numerical solution of hyperbolic PDE in one and two spaces dimension: explicit and implicit schemes: ADI methods. Differences schemes for first order equations, Numerical solutions of elliptical equations

## Unit IV

Approximations of Laplace Solutions of Dirichlet, Neuman and mixed type problems. Finite element methods: Linear, triangular elements and rectangular elements.

### Recommended Books:

1. M. K. Jain, S.R.K. Iyenger and R. K. Jain: Computational Methods for Partial differential equations, Wiley Eastern, 1994.
2. M. K. Jain, Numerical Solution Differential Equation, 2<sup>nd</sup> Edition, Wiley Eastern.
3. S. S. Sastry, Introductory Methods of Numerical analysis, Prentice-Hall of India, 2002.
4. D .V. Griffiths and I. M. Smith, Numerical Methods of Engineers, Oxford University Press, 1003.
5. C. F. General and P.O. Wheatley: Applied Numerical Analysis, Addison-Wiley, 1998.
6. B. S Grawal: Higher Engineering Mathematics , Khanna Publication.
7. J. N. Reddy: Introduction to Finite Element Method

### Course Outcomes:

On completion of the course the students

- CO 1. will be able to make intelligent choices of methods for specific problems
- CO 2. know the convergence conditions for the different methods
- CO 3. knows which order the different methods have
- CO 4. has knowledge of state of the art numerical method in the field
- CO 5. is able to use the methods in numerical calculations.

## Paper IV: Advanced Topology (MAT-307)

Total credit-5

**Course Objective:** The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand Characterization of connected sets in terms of open sets and closed sets, Directed sets, nets and subnets filter, filter base and subbase , Tychonoff product topology in terms standard subbase and its characterizations in terms of projection maps, continuous functions, Product of  $T_0, T_1, T_2$ , spaces, Connectedness and compactness, first and second countability for product spaces. Homotopy of paths.
3. To prepare the students for further research in analysis.

## Unit I

Characterization of connected sets in terms of open sets and closed sets. Closure of a connected set. Union of connected sets. Connected sets in  $\mathbb{R}$ . Continuity of a function and connectedness. Components and partition of space. Path connected space.

## Unit II

Inadequacy of sequential convergence. Directed sets, nets and subnets and their examples Convergence of a net, charaterisation of open sets, closed sets, closure, cluster point and limit point of a set in terms of net convergence. Hausdorffness and continuity of a function in terms of nets.

### Unit III

Definition of filter and its examples. Neighborhood filter. Comparison of filters. Filter base and subbase. Convergence of a filter. Ultrafilters. Continuous functions and filters. Net based on filter and filter based on net. Quotient topology, quotient space, quotient map, quotient space  $X/R$ , Finite product space, projection mapping.

### Unit IV

Tychonoff product topology in terms standard subbase and its characterizations in terms of projection maps, continuous functions, Product of  $T_0, T_1, T_2$ , spaces. Connectedness and compactness, first and second countability for product spaces. Homotopy of paths.

### Books Recommended:

1. George F. Simmons : Introduction to Topology and Modern Analysis, Mc Graw-Hill Book Company 1963.
2. J.L. Kelley : General Topology, Van Nostrand, Reinhold Co., New York 1995.
3. K.D. Joshi : Introduction to General Topology, Wiley Eastern Ltd., 1983.
4. James R Munkres : Topology, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
5. S. Willard : General Topology Addison-Wesley, Reading, 1970.

**Course Outcome:** After studying this course the student will be able to

- CO 1. understand the basic of this course and think & develop new ideas in this course.
- CO 2. know about Characterization of connected sets in terms of open sets and closed sets, Directed sets, nets and subnets filter, filter base and subbase , Tychonoff product topology in terms standard subbase and its characterizations in terms of projection maps, continuous functions, Product of  $T_0, T_1, T_2$ , spaces, Connectedness and compactness, first and second countability for product spaces. Homotopy of paths.
- CO 3. covers a wide area of research in analysis.

### Paper IV: Hydro Dynamics (MAT-308)

**Total Credit-5**

**Course Objectives:** To study Lagrange's stream function, Two dimensional source, sink, doublet, Theorem of Blasius, general motion of cylinder, ellipse and sphere, Stoke's function, Irrotational motion, Kelvin's proof of performance. Motion due to circular and rectilinear vertices.

### Unit I

Lagrange's Stream function. Irrotational motion in two-dimensions. Complex velocity potential. Two dimensional Sources and sinks. Complex potential due to a source. Doublets. Complex potential for doublets. Image. Image of a source and doublets in a circle.

## Unit II

Milne-Thomson circle theorem. Theorem of Blasius. General motion of the cylinder. Motion of a circular cylinder. The motion in the case of a liquid streaming past a fixed circular cylinder. Initial motion between two coaxial cylinders.

## Unit III

Kinetic energy of liquid. Flow and circulation. Motion of elliptic cylinders. Joukowski transformation. Streaming past a fixed elliptic cylinder. Velocity potential and stream function in case of elliptic cylinders. Circulation about an elliptic cylinder.

## Unit IV

Motion of a sphere through a liquid. Liquid streaming past a fixed sphere. Problem of initial motion of sphere. Stoke's stream function. Irrotational motion. Vortex motion. Vortex lines. Kelvin's proof of permanence. Motion due to circular and rectilinear vortices.

### Books Recommended:

1. J.K. Goyal and K.P. Gupta: Fluid Dynamics, Pragati Prakashan, Meerut, 2017
2. N. Curle and H. J. Davis: Modern Fluid Dynamics, D. Van Nostrand Company Ltd. London, 1968.
3. B.G. Verma: Hydrodynamics, Pragati Prakashan, Meerut, 1995.
4. G.K. Batchelor: An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, 2000.

### Course Outcomes:

- CO 1. After this study students will have better understanding of hydrodynamical motion which play a very important role in applied sciences and physical problems.
- CO 2. Hydrodynamics opens the wide door of research in applied mathematics and various physical problems.
- CO 3. This course is also useful in various competitive exams like IAS, PCS, Faculty exams of higher education etc.

## Paper V: Discrete Mathematics (MAT-309)

Total Credit-5

**Course Objective:** Discrete Mathematics is the study of mathematical structure that are countable otherwise distinct and separable. Concepts of discrete mathematics are useful in studying and describing abstract and problems in computer science

### Unit 1

Semigroups & Monoids: Definition and examples of Semigroups and Monoids. Homomorphism of Semigroups and Monoids. Congruence relation and Quotient Semigroups. Subsemigroup and Submonoids. Direct products. Basic homomorphism theorem.

### Unit II

Lattices: Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sublattices. Direct products and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices.

### **Unit III**

Boolean Algebras: Boolean Algebras as Lattices, Various Boolean Identities. The Switching Algebra example. Subalgebras. Direct Products and Homomorphisms. Join-irreducible elements, Atoms and Minterms. Boolean Forms and their Euivalence.

### **Unit IV**

Graph Theory: Definition of Graphs, Paths, Circuits, Cycles & Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar graphs and their properties. Trees. Euler's Formula for connected planar graphs.

### **Books Recommended:**

1. C.L. Liu: Elements of Discrete Mathematics (Second Edition), McGraw Hill, International Edition, Computer Science Series, 1986.
2. J.P. Tremblay & R. Manohar: Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
3. N. Dew. Graph Theory with Application to Engineering and Computer Sciences, Prentice Hall of India.

### **Course Outcomes:**

- CO 1. The discrete mathematics is modern trend in world-wise era of computer science especially Cryptography, Rational Databases, Logistics Computer algorithm, robotics, Google maps, etc.
- CO 2. It also can be used in Computer Networking in many Government and Private agency.

### **Paper V: Mathematical Modelling (MAT-310)**

**Total Credit-5**

### **Course Objectives:**

1. The overall objective of this course is to enable students to build mathematical models of real-world phenomenon, analyze them and make predictions about behavior of these systems.
2. Variety of modeling techniques will be discussed with real world problems.

### **Unit I**

Need, techniques, classification and simple illustrations of mathematical modelling. Limitations of mathematical modelling, Linear and Non-linear Growth and Decay models. Compartment models. Some techniques for analyzing ordinary differential equation mathematical models.

## Unit II

Continuous Models for interacting Population: Interaction between species: two species models, definition of stability, community matrix approach, Qualitative behavior of the community matrix, Lotka-Volterra model prey predator model, Models for mutualism and competition among the species.

## Unit III

Mathematical modelling in epidemiology, basic concepts, SI model, SIS model with constant coefficient, SIS model when coefficients are function of time, SIS model with constant number of carriers, General deterministic model with removal (Kermack McKendrick model), Epidemic model with vaccination.

## Unit IV

Some simple drug distribution problems, mathematical modelling in pharmacokinetics, the distribution of metabolites in the body, physiological application of the two compartment model, mathematical modeling of drug effects- a more general approach.

### Books Recommended:

1. J.N. Kapur: Mathematical Modelling, New Age International (P) Limited, New Delhi.
2. Zafar Ahsan : Differential Equations and Their Applications, PH I learning Private Limited, New Delhi.
3. J. Mazumdar: An Introduction to Mathematical Physiology and Biology, Cambridge University Press.
4. Nicholas F. Britton: Essential Mathematical Biology, Springer.

### Course Outcomes:

On successful completion of the course, a student is able to

- CO 1. assess and articulate what type of modeling techniques are appropriate for a given system,
- CO 2. construct mathematical models in population biology, epidemics, pharmaco- kinetics etc and analyze it,
- CO 3. make predictions of the behavior of a given system based on the analysis of its mathematical model.

## Paper IV: Complex Manifolds (MAT-311)

**Total Credit-5**

**Course Objective:** The paper of Complex Manifolds is introduced to M.Sc. classes for the study of Almost Complex Manifolds, F-connection, half symmetric connection, Almost Hermit Manifolds, Kaehler Manifolds and Nearly Kaehler Manifolds, Curvature identities, almost analytic vectors. The main objective of Complex Manifolds is that to prepare the students for further research in analysis of differential geometry and structure of differentiable manifold.

## Unit I

Almost Complex Manifolds: Elementary notions, Nijenhuis tensor Eigen values of F, Integrability conditions, Contravariant and covariant analytic vectors, F-connection, half symmetric connection.

## Unit II

Almost Hermit Manifolds: Definition, Almost analytic vector fields. Curvature tensor. Linear connections.

## Unit III

Kaehler Manifolds: Definition. Curvature tensor. Affine connection. Properties of projective, conformal, concircular and conharmonic curvature tensors. Contravariant almost analytic vector

## Unit IV

Nearly Kaehler Manifolds: Introduction, Curvature identities, almost analytic vectors.

### Books Recommended:

1. R.S. Mishra: Structure on differentiable manifold and their application, ChandramaPrakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structures of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.
3. U.C.De and A.A.Shaaikh, Complex and Contact Manifolds, Narosa Publishing House, New Delhi 2009.

### Course Outcome:

- CO 1. This paper is to investigate structure of differentiable manifold.
- CO 2. Students will be able to demonstrate an intuitive and computational understanding of Differentiable manifold, Riemannian Manifold, Almost Complex Manifolds, F-connection, half symmetric connection, Almost Hermit Manifolds, Kaehler Manifolds and Nearly Kaehler Manifolds.
- CO 3. Complex Manifolds covers a wide area of research in differential geometry. It is also used in General Relativity and Gravitation and Cosmology.

### Paper V: Riemannian Geometry (MAT-312)

**Total Credit-5**

**Course Objective:** The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand Generalised covariant differentiation, Gauss's formulae, Curvature of a curve in a hypersurface, Normal curvature, Mean curvature, Change from one set of normal to another. Curvature of a curve in subspace, Infinitesimal transformation, Hyperplanes, Hyperspheres, Geodesics in a space.
3. To prepare the students for further research in differential geometry & Riemannian Geometry.

## Unit 1

Unit normal. Generalised covariant differentiation. Gauss's formulae. Curvature of a curve in a hypersurface. Normal curvature. Mean curvature. Principal normal curvature. Lines of curvature. Conjugate and asymptotic directions. Tensor derivative of the unit normal. Gauss characteristic equation and Mainardi-Codazzi equations. Totally geodesic hypersurfaces.

## Unit II

Unit normals. Gauss's formulae. Change from one set of normals to another. Curvature of a curve in subspace. Conjugate and asymptotic directions. Generalisation of Dupin's theorem. Derived vector of a unit normal. Lines of curvature for a given normal.

## Unit III

Infinitesimal transformation. The notion of Lie derivative. Lie derivative of metric tensor and connection. Motion and affine motion in Riemannian spaces.

## Unit IV

Hyperplanes. Hyperspheres. Central quadric hypersurfaces. Reciprocal quadric hypersurfaces. Conjugate radii. Any hypersurface in Euclidean spaces. Riemannian curvature of a hypersphere. Geodesics in a space of positive constant curvature. (1 credit)

### Books Recommended:

1. C.E. Weatherburn: An Introduction to Riemannian Geometry and the Tensor Calculus, Cambridge University Press, 1966.
2. K. Yano: The Theory of Lie Derivatives and its Applications, North Holland Publishing Company, Amsterdam, 1957.
3. R. S. Mishra: A Course in Tensors with Applications to Riemannian Geometry, Pothishala (Pvt.) Ltd., 1965.

**Course Outcome:** After studying this course the student will be able to

- CO 1. understand the basic of this course and think & develop new ideas in this course.
- CO 2. know Generalised covariant differentiation, Gauss's formulae, Curvature of a curve in a hypersurface, Normal curvature, Mean curvature, Change from one set of normals to another. Curvature of a curve in subspace, Infinitesimal transformation, Hyperplanes, Hyperspheres, Geodesics in a space.
- CO 3. cover a wide area of research in differential geometry & Riemannian Geometry.

### Paper V: Magneto Hydrodynamics (MAT-313)

**Total Credit- 5**

**Course Objective:** The main objectives are

1. Train the student to communicate mathematical ideas in a lucid and effective manner.

2. To understand Maxwell equation, Electromagnetic field in a conductor, Rate of flow of charge, Alfven theorem, basic concept of viscous and Viscous magnetohydrodynamics Alfven wave, Parallel steady flow, Hartmann and Couette flow.
3. To prepare the students for further research in magnetohydrodynamics.

### **Unit I**

Maxwell equations. Electromagnetic field in a conductor. MHD approximations. Rate of flow of charge. Important MHD parameters. Diffusion of magnetic field. Frozen-in-fields. Integral of magnetic field equation. Analogy of magnetic field with vorticity

### **Unit II**

Alfven theorem. Lorentz force and its transformations. Magnetic energy. Poynting vector theorems. Basic equations of in viscous and viscous magnetohydrodynamics. Energy conservation law.

### **Unit III**

Alfven waves. MHD waves in a compressible fluid. Equi-partition of energy of Alfven waves. MHD boundary conditions. Equations of incompressible MHD flow.

### **Unit IV**

Parallel steady flow. Steady parallel flow in a conservative field of force. One-dimensional steady viscous MHD flow. Hartmann flow. Couette flow.

### **Books Recommended:**

1. Alan Jeffery: Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966.
2. F. Chorlton: Text Book on Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
3. S.I. Pai: Magnetohydrodynamics and Plasma Dynamics, Springer-Verlag, 1962.
4. P. A. Davidson: Introduction to Magnetohydrodynamics, Cambridge University Press, U.K., 2017.

**Course Outcomes:** After studying this course student will be able to

- CO 1. understand the basic of this course and think and develop in this course.
- CO 2. know about Maxwell equation, Electromagnetic field, basic concept of viscous and viscous MHD.
- CO 3. cover a wide area of research in MHD.

## M.A/M.Sc. Fourth Semester Based on CBCS (Mathematics)

**Paper I: Measure Theory (MAT-401)**

**Total Credit-5**

**Course Objectives:** A measure is a generalization of the concept of length, area and volume.

### **Unit I**

Lebesgue's outer measure and its properties. Length of an interval and Lebesgue outer measure. Lebesgue measurable sets in  $\mathbb{R}$  and  $\sigma$ -algebra of Lebesgue measurable sets in  $\mathbb{R}$  Lebesgue measurability of open sets, closed sets and Borel sets. Lebesgue measure on  $\mathbb{R}$ . Example of a Non-Lebesgue measurable set. Cantor's set and its Lebesgue measure. General outer measure. Caratheodory's definition of measurable sets.  $\sigma$ -algebra of measurable sets.

### **Unit II**

Definition of a measure. Measurable space and a measure space. Definition of a measurable function. Equivalent conditions for measurable function. Sum and product of measurable functions. Composition of a measurable and a continuous function. Sequences of measurable functions. Measurability of supremum function, infimum function, limit superior function, limit inferior function and limit function.

### **Unit III**

Simple measurable functions and their properties. A non-negative measurable function as the limit of a sequence of non-negative simple measurable functions. Concept of almost everywhere (a.e.). Lebesgue theorem. Convergence in Measure and its properties. F. Riesz theorem and Egorov theorem. Convergence almost everywhere, almost uniform convergence and their inter-relations.

### **Unit IV**

Lebesgue Integration of a simple measurable function on  $\mathbb{R}$  and its properties. Lebesgue Integration of a bounded measurable function on a set  $E$  with finite Lebesgue measure, i.e.  $\lambda(E) < \infty$  and its properties. Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, Lebesgue integration and Riemann integration. Integration on a measure space. Lebesgue integral of general measurable function and its properties.

### **Books Recommended:**

1. Walter Rudin, Principle of Mathematical Analysis (3rd edition) McGraw-Hill Kogakusha, International Student Edition, 1976.
2. P. R. Halmos, Measure Theory, Van Nostrand, 1950.
3. G. de Barra, Measure Theory and Integration, Wiley Eastern, 1981.
4. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International, New Delhi, 2000.
5. R. G. Bartle, The Elements of Integration, John Wiley, 1966

## Course Outcomes:

- CO 1. Measure theory has an application in Statistics because measure theoretic probability to be very useful in helping to understand deeper issue in probability theory.
- CO 2. Measure theory has an application in economics also.

## Paper II: Hilbert Spaces (MAT-402)

Total Credit-5

**Course Objective:** Hilbert space is an abstract vector space possessing the structure of an inner product that allows length and angle to be measured.

### Unit I

Inner product spaces, their basic properties and examples, Schwartz inequality. Norm induced by inner product, Continuity of inner product, Hilbert spaces and their examples. Parallelogram equality, polarization identity. Characterization of inner product in terms of norm. Separable Hilbert spaces and their examples.

### Unit II

Orthogonal vectors. Orthogonal complement. Projection theorem. Projection operators. Orthogonal sets and their advantage over its linearly independent sets. Complete orthonormal sets. Bessel's generalized inequality. Parseval's Relation

### Unit III

Grahm-Schmidt orthogonalization process. Fourier series representation. Bounded linear functionals on Hilbert spaces. Riesz-Frechet representation theorem. Dual spaces. Inner product structure of dual spaces. Reflexivity of Hilbert spaces.

### Unit IV

Hilbert adjoint operators. Shift operators. Special cases of Hilbert adjoint operators self adjoint operators, positive operator, normal operators, unitary operators. Orthogonal projection operators. Eigen Values of Linear Operator. Spectrum of a Bounded Linear Operator. Spectral properties of bounded linear operators.

### Books Recommended:

1. P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.
2. B. Choudhary and S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
3. I.J Maddox: Functional Analysis, Cambridge University Press (1970).
4. B.V.Limaye: Functional Analysis, New Age International Publications, New Delhi.
5. K. Chandrashekara Rao. Functional Analysis, Narosa Publishing House, New Delhi
6. W.Rudin: Functional Analysis, TMH, New Delhi
7. H.K.Pathak: Functional Analysis with Applications, Siksha Sahitya Prakashan, Merru

### Course Outcomes:

- CO 1. Hilbert spaces arise naturally and frequently in Mathematics and Physics typically as infinite dimensional function space.
- CO 2. It is used in solving partial differential equations and Fourier analysis, etc.

### Paper III: Analytical Dynamics (MAT-403)

Total Credit-5

### Course Objectives:

1. To provide a simple and general description of the system dynamics which is invariant under coordinate transformations?
2. To give knowledge of general coordinate system to the students and its significance.
3. To provide an introduction of Lagrangian and Hamiltonian approach, small oscillations and canonical transformations.

### Unit I

Classification of dynamical systems, generalized coordinates, Holonomic and non holonomic systems, Kinetic energy, Generalized components of momentum, Generalized Components of the effective and applied forces, Lagrange's equations, Some Examples on Lagrange's equations, Energy equation from Lagrange's equation, Reciprocal relations, Ignorance of coordinates, The Routhian function, Lagrange's equation for impulsive motion, Euler's equation from Lagrange's equation.

### Unit II

Small oscillations, Lagrange's equation of small oscillations. Lagrange's determinants, Normal modes and normal coordinates and their stationary properties.

### Unit III

Hamilton's equations of motion. Applications of Hamiltonian method, Action, Hamilton's principle, Principle of least action, Hamilton-Jacobi equation, Hamilton-Jacobi theorem.

### Unit IV

Canonical transformations, conditions of canonicity, cyclic relations, Generating function, Phase space, Bilinear invariants, Poisson brackets, Lagrange brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

### Books Recommended:-

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and a Rigid Body, Macmillan India Ltd., 1982.
2. A.S. Ramsey: Dynamics part-II, The English Language Book Society and Cambridge University Press, 1972.
3. J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book company, 1982
4. P.P. Gupta, G.S. Malik: Rigid Dynamics I, Krishna Prakashan Media (P) Ltd. 2008.

5. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi.

### Course Outcomes:

- CO 1. Students will get ability to write Kinetic energy, angular momentum etc in terms of generalized coordinates.
- CO 2. The students will be able to understand the Lagrangian and Hamiltonian formalisms of the laws of motion, which are generalizations of Newton's equations of motion.
- CO 3. Students will be able to apply the formalisms of analytical dynamics to practical examples of small oscillations.
- CO 4. Students will be able to understand the notion of phase space and Liouville's theorem.
- CO 5. Students will acquire an ability to learn the related mathematical methods such as coordinate transformations and variational methods, and to apply the formalisms of analytical dynamics to real life problems.

### Paper IV: Fixed Point Theory and its Applications (MAT-404)

Total Credit-5

**Course Objective:** A fixed point theory is a collection of results saying that a function  $F$  will have at least one fixed point (a point  $x$  for which  $F(x)=x$ ) under some conditions on function. Fixed point has been revealed as a very powerful and important tool in the study of nonlinear phenomena.

#### Unit I

Background of Metrical fixed point theory, Fixed Points, Uniformly convex, strictly convex and reflexive Banach spaces, Lipschitzian and contraction mapping, Banach's contraction principle, Application to Volterra and Fredholm integral equations, Caristi's fixed point theorem.

#### Unit II

Nonexpansive, asymptotically nonexpansive and quasinonexpansive mappings and Fixed Points, Fixed point theorems for nonexpansive mappings, Nonexpansive operators in Banach spaces satisfying Opial's conditions, The demiclosedness principle.

#### Unit III

Brouwer's fixed point theorem, Schauder's fixed point theorem, Measure of Non- Compactness, Condensing map, Fixed points for condensing maps, Strict convexity, Uniform convexity, The modulus of convexity and normal structure, Smoothness, retraction map, Sadovskii's fixed point theorem, Introduction of Set-valued mappings, Set-valued contraction map, Hausdorff metric, Nadler's fixed point theorem.

#### Unit IV

Fixed point iteration procedures: Krasniselskij iteration, Picards iteration, The Mann Iteration, Lipschitzian and Pseudocontractive operators in Hilbert spaces, Strongly pseudocontractive operators in Banach spaces, The Ishikawa iteration, Equivalence between Mann and Ishikawa iterations.

### Books Recommended:

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.
2. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, New York, 2001.
3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.
4. V. I. Istratescu, Fixed Point Theory, An Introduction, D. Reidel Publishing Co., 1981. 5. K. Goebel and W. A. Kirk, Topic in Metric Fixed Point Theory, Cambridge University Press, 1990.

### Course Outcomes:

- CO 1. It is used in finding unique solution of differential and integral equations.
- CO 2. It has also applications in biology, chemistry, economics, game theory, engineering and physics.

### Paper IV: Cosmology (MAT-405)

**Total Credit-5**

**Course Objectives:** Cosmology is a branch of science in which we study the evolution of universe through an construct the cosmological models.

#### Unit I

Static cosmological models, Einstein Universe, de-Sitter Universe, Hubble law, Weyl Postulate

#### Unit II

Non-Static cosmological models, Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological models and its properties, Observable parameters in FRW metric, Particles Horizon, Event Horizon,

#### Unit III

Einstein's field equation and dynamics of the universe. Gravitational Collapse, Gravitational Collapse of a Homogeneous Dust ball, Black Holes (Strong Gravitational fields),

#### Unit IV

The Kerr metric or the Rotating black Holes, Origin and Evolution of Universe, Creation of matter, C Field Theory, Explosive Creation, Steady State Theory.

### Books Recommended:-

1. K. D. Krori : Fundamentals of Special and General Relativity; PHI Publication, 2010.
2. S. R. Roy and Raj Bali : Theory of Relativity; Jaipur Publishing House, 2008.
3. Steven Weinberg : Gravitation and Cosmology : Principles and applications of General Relativity; Wiley Publ.,2005.
4. J. V. Narlikar : An Introductions to Relativity; Cambridge University Press, 2010.
5. I.B. Khriplovich : General Relativity; Springer Science + business media, 2005.

**Course Outcomes:**

- CO 1. The studies of evolution of universe describe the origin, recent scenario and future of our universe.
- CO 2. Recent outcomes of cosmological studies predict that our universe is expanding and accelerating which further manifest that our universe contains 96% of invisible matter of exotic nature. The recent physics Nobel prize (2019) were awarded in cosmology field.

**Paper IV: Wavelet Analysis (MAT-406)****Total Credit-5****Course Objectives:** The main objectives are:

1. To study Fourier and inverse Fourier transforms convolution and delta function, Fourier transform of square integrable functions.
2. To study wavelet transform, time frequency Analysis, Gabor transform, Dyadic wavelets and inversion.
3. To study frames, Wavelet series, Scaling functions and wavelets, Multi resolution analysis.
4. To study compactly supported wavelets and their duals, orthogonal wavelets and wavelet Backet, Examples of orthogonal wavelets, orthogonal wavelet packets, orthogonal decomposition of wavelet series.

**Unit I**

Fourier Analysis: Fourier and inverse Fourier transforms, Convolution and delta function, Fourier transform of Square integrable functions. Fourier series, Poisson's Summation formula.

**Unit II**

Wavelet Transforms and Time Frequency Analysis: The Gabor Transform. Short-time Fourier transforms and the uncertainty principle. The integral wavelet transforms Dyadic wavelets and inversions.

**Unit III**

Frames. Wavelet Series. Scaling Functions and Wavelets: Multi resolution analysis, scaling functions with finite two scale relations. Direct sum decomposition of  $L^2(\mathbb{R})$ : Linear phase filtering.

**Unit-IV**

Compactly supported wavelets, Wavelets and their duals, Orthogonal Wavelets and Wavelet packets, Example of orthogonal Wavelets. Identification of orthogonal two-scale symbols, Construction of Compactly supported orthogonal wavelets, Orthogonal wavelet packets, orthogonal decomposition of wavelet series.

**Recommended Books:**

1. C. K. Chui, A First Course in Wavelets, Academic press NY 1996.
2. Daubechies, Ten Lectures in Wavelets, Society for Industrial and Applied Maths, 1992.

**Course Outcomes:** After the course the students will be able to,

CO 1. understand the basics of this course.

CO 2. to think and develop new ideas in this field.

CO 3. to understand the use of this course in science and technology and other fields of Mathematical Analysis.

#### **Paper IV: Hydro Statics (MAT-407)**

**Total Credit-5**

**Course Objective:** The main objectives are

1. Train the student to communicate mathematical ideas in a lucid and effective manner.
2. To understand fluid pressure, surface of equal pressure, elastic fluids, rotating fluids. The equilibrium of floating body etc.
3. To prepare the student for further research in Hydrostatics.

#### **Unit I**

Fluid Pressure: Equation of pressure, Necessary condition of equilibrium, surface of equal pressure, curves of equal pressure and density, Elastic fluids, Rotating fluids.

#### **Unit II**

Resultant Pressure and Centre of pressure: Formula for Centre of pressure, geometrical position of centre of pressure, locus of centre of pressure, resultant pressure on curved surfaces.

#### **Unit III**

The equilibrium of Floating Bodies: Conditions of equilibrium, Principle of potential energy and Work done, Surface of Buoyancy.

#### **Unit IV**

Stability of Floating Bodies: Meta centre, Conditions of stability, Work done in small displacement, floating vessel containing liquid, Stability in Heterogeneous Liquid.

#### **Books recommended:**

1. Bhu Dev Sharma: Hydro – statics, Kedar Nath Ram Nath.
2. M. Ray, H. S. Sharma : A Text Book of Hydro- statics, S. Chand
3. Rahman: Hydrostatics, Savera Publishing House.
4. N. Inoue, M. Nishihara: Hydrostatic Extrusion: Theory and Applications, Springer
5. S.L. Loney: Mechanics and Hydrostatics for beginners, Cambridge University Press

**Course Outcomes:**After studying this course the student will able to

CO 1. understand the basic of this course and think and develop in this course.

CO 2. know about Fluid pressure, Elastic fluids, Rotating Fluids, Resultant pressure and Center of Pressure, the equilibrium of Floating bodies, Metacentre etc.

CO 3. Cover a wide area of research in Hydrostatics.

**Paper IV: Mathematics for Humanities (MAT-408)****Total Credit-5**

**Course Objectives:** The main objective of this course is to make students aware of the fundamentals of Mathematics and to help them to explore applications of Mathematics in daily life.

**Unit I**

Functions and graphs, Elementary functions, Exponential functions and Natural Logarithms, Trigonometrical functions, Limit and continuity of Functions.

**Unit II**

The meaning of derivative, Calculus of derivative, Applications of derivative, Optimization, Exponential Growth and Decay, Linearization of Functions.

**Unit III**

Functions of two variables, Graphical representation of functions, Linearization of functions of two variables, Vectors and matrices, System of linear equations, The inverse matrix.

**Unit IV**

First order ordinary differential equations, Applications of ordinary differential equation in exponential growth and decay, Solutions and direction fields, Ordinary differential equation with variables separable. The spread of infectious diseases, Drug dosage.

**Books Recommended:**

1. Shaffer Hall: Differential and Integral calculus with Applications, MEDTECH, Scientific International Pvt. Ltd.
2. Arun Kumar: Mathematics for Biologists, Narosa Publishing House, New Delhi

**Course Outcomes:**

- CO 1. On completion of this course, students will be able to understand basics of Mathematics and use Mathematics as a tool to study real world problems and analyse them.

**Paper V: Information Theory (MAT-409)****Total Credit-5**

**Course Objective:** The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand Measure of Information-Axiom for a measure of uncertainty, Discrete Memoryless Channel-Classification of channels, Information processed by a channel, Some intuitive properties of a measure of entropy Symmetric, normalization, expansibility, boundedness, recursivity, maximality, stability, additivity, subadditivity, nonnegativity, continuity, branching etc. , The general solution of fundamental equation of equation of information.
3. To prepare the students for further research in applied mathematics.

## Unit I

Measure of Information-Axiom for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Transformation and its properties. Noiseless coding-Ingredients of noiseless coding problem. Uniquely decipherable codes. Necessary and sufficient condition for the existence of instantaneous codes. Construction of optimal codes.

## Unit II

Discrete Memoryless Channel-Classification of channels. Information processed by a channel. Calculation of channel capacity. Decoding schemes. The ideal observer. The fundamental theorem of information theory and its strong and weak converses. Continuous Channels-The time-discrete Gaussian channel. Uncertainty of an absolutely continuous variable. The converse of the coding theorem for time-discrete Gaussian channel. The time-continuous Gaussian channel. Band-limited channels.

## Unit III

Some intuitive properties of a measure of entropy Symmetric, normalization, expansibility, boundedness, recursivity, maximality, stability, additivity, subadditivity, nonnegativity, continuity, branching etc. and interconnections among them. Axiomatic characterization of the Shannon entropy due to Shannon and Fadeev. Information functions, the fundamental equation of information, information functions continuous at the origin, nonnegative bounded information functions, measurable information functions and entropy. Axiomatic characterisations of the Shannon entropy due to Tverberg and Leo.

## Unit IV

The general solution of fundamental equation of equation of information. Derivations and their role in the study of information functions. The branching property. Some characterizations of the Shannon entropy based upon the branching property. Entropies with sum property. The Shannon inequality. Subadditive, additive entropies. The Renji entropies. Entropies and mean values. Average entropies and their equality, optimal coding and the Renji entropies. Characterization of some measures of average code length.

### Books Recommended:

1. R. Ash, Information Theory, Interscience Publishers, New York, 1965.
2. F.M. Reza, An introduction to Information Theory, McGraw-Hill Book Company Inc., 1961.
3. J. Aczel and Z. Daroczy, On measures of information and their characterizations, Academic Press, New York.

**Course Outcome:** After studying this course the student will be able to

- CO 1. understand the basic of this course and think & develop new ideas in this course.
- CO 2. know Measure of Information-Axiom for a measure of uncertainty, Discrete Memoryless Channel-Classification of channels, Information processed by a channel, Some intuitive properties of a measure of entropy Symmetric, normalization, expansibility, boundedness, recursivity, maximality, stability, additivity, subadditivity, nonnegativity, continuity, branching etc. , The general solution of fundamental equation of equation of information.
- CO 3. cover a wide area of research in applied mathematics.

**Course Objectives:**

1. To introduce students to the application of mathematical modelling in the analysis of biological systems.
2. To show how mathematics and computing can be used in an integrated way to analyse biological systems.

**Unit I**

Introduction, Definition and scope of Bio-Mathematics, Role of Mathematics in Biosciences. Bio-Fluid Dynamics, Human Cardiovascular System and Blood flows, Blood flow through artery with mild stenosis. Two layered flow in a tube with mild stenosis.

**Unit II**

Pulsatile flow of blood, analysis and applications of arterial flow dynamics, derivation of aortic Diastolic- Systolic pressure waveforms, Moens-Korteweg expression for pulse wave velocity in an inviscid fluid filled elastic cylindrical arterial tube model, Analysis and applications of left ventricular mechanics, analysis and applications of heart valve vibration.

**Unit III**

Human Respiratory System, Gas exchange and air flow in human lungs. Consumption and transport of Oxygen. Weibel's model for flows in human lung airways, Comparison between flows of blood and flows in lung airways.

**Unit IV**

Diffusion, Fick's laws of diffusion, Diffusion equation and its solution, Modification of the diffusion equation, Diffusion in artificial kidney, Hemodialyser, Types of Hemodialyser.

**Books Recommended:**

1. J.N. Kapur: Mathematical Models in Biology and Medicine, Affiliated East-West Press Pvt. Ltd., New Delhi, 1985.
2. Y.C. Fung: Bio-Mechanics, Springer-Verlag New York Inc., 1990.
3. Stanley E. Charm and George S. Kurland: Blood Flow and Microcirculation, John Wiley & Sons, 1974.
4. S.A. Levin: Frontiers in Mathematical Biology, Springer-Verlag, 1994.
5. S.K. Pundir & R. Pundir : Biomathematics, Pragati Prakashan, 2010.
6. J. Mazumdar: An Introduction to Mathematical Physiology and Biology, Cambridge University Press.

**Course outcomes:**

At the end of the course, students will be able to

- CO 1. have an enhanced knowledge and understanding of mathematical modelling and statistical methods in the analysis of biological systems,
- CO 2. have sound knowledge of developing mathematical models in the areas of Bio sciences and bio fluid dynamics and their analysis.
- CO 3. develop skills in algebraic manipulation, the calculus of linear and non-linear differential equations.

**Paper V: Contact Manifolds (MAT-411)**

**Total Credit 5**

**Course Objective:** The paper of Contact Manifolds is introduced to M.Sc. classes for the study of Almost Contact Manifolds, Normal contact structure, Affinily almost cosymplectic manifold, Almost Grayan Manifolds, Sasakian Manifolds, Properties of projective, conformal curvatures in Sasakian manifold, Concircular and con- harmonic curvatures in Sasakian manifold. Cosymplectic structure, Nearly Cosymplectic structure and F-structure manifolds. The main objective of Contact Manifolds is that to prepare the students for further research in analysis of differential geometry and structure of differentiable manifold.

**Unit I**

Almost Contact Manifolds: Definition. Eigen values of F. Intergrability conditions of  $\pi_m$ ,  $\pi_m$  and  $\pi_1$ . Lie derivative. Normal contact structure. Affinily almost cosymplectic manifold.

**Unit II**

Almost Grayan Manifolds: Introduction. D-conformal transformation. Particular affine connections. Almost Sasakian manifold. Quasi-Sasakian manifold.

**Unit III**

Sasakian Manifolds: K-contact Riemannian manifold and its properties, Sasakian manifold and its properties. Properties of projective, conformal curvatures in Sasakian manifold.

**Unit IV**

Concircular and con- harmonic curvatures in Sasakian manifold. Cosymplectic structure and Nearly Cosymplectic structure. F-structure manifolds: Definitions and some basic properties.

**Books Recommended:**

1. R.S. Mishra: Structure on differentiable manifold and their application, ChandramaPrakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structures of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984

**Course Outcome:**

- CO 1. This paper is to investigate structure of differentiable manifold.
- CO 2. Students will be able to demonstrate an intuitive and computational understanding of Differentiable manifold, Riemannian Manifold, Almost Contact Manifolds, Normal contact structure, Affinily almost cosymplectic manifold, Almost Grayan Manifolds, Sasakian Manifolds, Properties of projective, conformal curvatures in Sasakian manifold,

Concircular and con-harmonic curvatures in Sasakian manifold. Cosymplectic structure, Nearly Cosymplectic structure and F-structure manifolds.

CO 3. Contact Manifolds covers a wide area of research in differential geometry. It is also used in General Relativity and Gravitation and Cosmology.

**Paper V: Finsler Geometry (MAT-412)**

**Total Credit-5**

**Course Objective:** The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand Finsler metric function, Dual tangent space, Geodesics, Fundamental postulates of Cartan, Berwald's covariant derivative and its properties, Commutation formula resulting from partial  $\delta$ -differentiation, Three curvature tensors of Cartan.
3. To prepare the students for further research in differential geometry & Finsler Geometry.

**Unit I**

Finsler metric function. Its properties. Tangent space. Indicatrix. Metric tensor and C-tensor  
Homogeneity properties of  $g_{ij}$  and  $C_{ijk}$ .

**Unit II**

Dual tangent space. Geodesics.  $\delta$ -differentiation. Partial  $\delta$ -differentiation. Properties of partial  $\delta$ -differentiation.

**Unit III**

Fundamental postulates of Cartan. Cartan's covariant derivatives and their properties. Geometry of paths. Berwald's covariant derivative and its properties.

**Unit IV**

Commutation formula resulting from partial  $\delta$ -differentiation. Other commutation formulae.  
Three curvature tensors of Cartan. Identities satisfied by curvature tensors including Bianchi identities.

**Books Recommended:**

1. H. Rund: The Differential Geometry of Finsler Spaces, Springer-Verlag, 1959.
2. M. Matsumoto: Foundations of Finsler Geometry and special Finsler spaces, Kaiseisha Press, Saikawa, Otsu, 520 Japan, 1986.

**Course Outcome:** After studying this course the student will be able to

- CO 1. understand the basic of this course and think & develop new ideas in this course.
- CO 2. know Finsler metric function, Dual tangent space, Geodesics, Fundamental postulates of Cartan, Berwald's covariant derivative and its properties, Commutation formula resulting from partial  $\delta$ -differentiation, Three curvature tensors of Cartan.
- CO 3. cover a wide area of research in differential geometry & Finsler Geometry.

**Course Objective:**The main objectives are

1. Train students to communicate mathematical ideas in a lucid and effective manner.
2. To understand the simple epidemic and SIS diseases, SIR epidemics, SIR endemics, Modelling AIDS epidemic, Tumor modelling, Phenomenological models, Human genetics, Basic models for inheritance, Genetic matrices, Hardy-Weinberg law, Models for Genetic Improvement.
3. To prepare the students for further research in applied mathematics and bio-mathematics.

### **Unit I**

The simple epidemic and SIS diseases, SIR epidemics, SIR endemics: No disease related death including disease related death, eradication and control, Vector-borne diseases, Basic model for macro parasitic diseases.

### **Unit II**

Modelling AIDS epidemic, Anderson's first model, Anderson's improved model, Interaction of HIV and Immune system, Stages in the course of HIV infection, Treatment of HIV infection, Modelling of HIV immunology, Analysis of treatment of HIV infection.

### **Unit III**

Tumor modelling, Phenomenological models, Nutrients, Diffusion limited stages, Moving boundary problem, Growth promoters and inhibitors, Vascularizations, Metastasis, Immune system response.

### **Unit IV**

Human genetics, Basic models for inheritance, Genetic matrices, Hardy-Weinberg law, Correlation between Genetic composition of siblings, Phenotype ratios, Multiple alleles and Application to Blood Group, Inheritance of sex linked characteristics, Models for Genetic Improvement: Selection and Mutation.

### **Books Recommended:**

1. J. Mazumdar: An Introduction to Mathematical Physiology and Biology, Cambridge University Press.
2. Nicholas F. Britton: Essential Mathematical Biology, Springer
3. J. N. Kapur: Mathematical Models in Biology and Medicine, Affiliated East-west Press Pvt. Ltd., New Delhi.
4. Fred Brauer, Carlos Castillo-Chavez: Mathematical Models in Population Biology and Epidemiology, Springer
5. Matt J. Keeling and Pejman Rohani: Modelling Infectious Diseases in Humans and Animals, Princeton University Press

**Course Outcome:** After studying this course the student will be able to

- CO 1. Understand the basic of this course and think & develop new ideas in this course.
- CO 2. Know The simple epidemic and SIS diseases, SIR epidemics, SIR endemics, Modelling AIDS epidemic, Tumor modelling, Phenomenological models, Human genetics, Basic models for inheritance, Genetic matrices, Hardy-Weinberg law, Models for Genetic Improvement.
- CO 3. Covers a wide area of research in applied mathematics and bio-mathematics.

